ON THE PRACTICAL USE OF GRAPHICAL PREDICTION METHODS[1]

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ABSTRACT

Useful forecasts may be obtained by graphical integrations of the dynamical prediction equations for a barotropic and a two-level baroclinic atmosphere. Such forecasts may be prepared without the aid of special equipment and are therefore particularly valuable as a means of training forecasters in physical prognosis.

The present paper reviews the physical principles, modeling assumptions, and methods of solution used in graphical prediction and introduces a method of obtaining surface forecasts which is considerably faster and simpler than previous methods. The predicted surface pressure is shown to be the sum of two components: (1) the pressure advected to the spot by one-half the 500-mb. wind and (2) a pressure change reflected down from aloft (actually one-half the 500-mb. height change expressed in equivalent pressure units at 1000 mb.). The movement of surface pressure systems is thus seen to be largely dependent on upper-level steering, while the deepening is found to be related to the vorticity advection at high levels, since this mainly determines the 500-mb. height changes.

Twenty sample surface forecasts prepared by the graphical method during July 1959 are presented and compared with the forecasts for the same dates issued by the National Weather Analysis Center. Little difference in accuracy is apparent.

Typical shortcomings and failures of the graphical prognoses are discussed. It is believed that the most serious errors are due to the use of only the initial 500-mb.charts in advecting the pressure systems. If the 500-mb. forecasts had been available earlier, it appears that a significant increase in accuracy could have been achieved by using both initial and forecast 500-mb. contours in performing the advections.

1. INTRODUCTION

The purpose of this paper is first to present a fast and simple method for preparing surface prognostic charts based on the graphical methods introduced by Estoque [1] and Reed [2], second, to show by examples the generally useful caliber of the forecasts obtained by the method, and third, to discuss some of the characteristic failures of the forecasts and possible ways of overcoming them. Graphical prediction, as referred to here, is a form of numerical prediction in which the dynamical equations are integrated by graphical operations rather than by machine computation. The technique of graphical integration was originated by Fjørtoft [3].

From the results presented in section 6 it will appear that the graphical method has possible useful application in the field today. However, it is felt that the principal advantage of the method lies not in its practical applications but rather in the physical insights it offers the forecaster regarding the movement and development of pressure systems and in the better appreciation it gives him of the dynamical approach to forecasting. Empirical rules, such as the steering of surface systems by the upper-level flow and the displacement of 500-mb. features by the space-mean flow, are shown to have a sound physical basis, and dynamical formulas or techniques of limited or overlapping scope, such as the Rossby wave formula and constant absolute vorticity trajectories, are brought within a single, broad framework.

The value of a unified, as opposed to a piecemeal, approach to the forecast problem is particularly apparent in the case of the student forecaster. The experienced forecaster may profitably include a variety of techniques in his "bag of tricks," though even in his case there is something to be said in favor of a unified outlook.

2. THE EQUATIONS AND THEIR LIMITATIONS

Before proceeding to the practical forecast procedure, it is desirable to present the basic equations and to recognize their limitations. For the 500-mb. forecast, the prediction equation is [3]

$$\frac{d}{dt}\left(\overline{Z}_5 - Z_5 + G\right) = 0, \tag{1}$$

where Z_5 is the geopotential height of the 500-mb. surface, $\overline{Z}_5$ is the space-mean 500-mb. height, usually measured at the corners of a square grid of 600–1000 km. mesh size, and G is a measure of the Coriolis parameter. The equation is essentially a statement of the conservation of absolute vorticity, and therefore assumes that a level of nondivergence exists in the atmosphere (near 500 mb.).

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Geostrophic motion is also assumed. Since absolute vorticity is conserved, it is apparent that this equation is unable to predict development or intensification.

The additional equation needed to extend the prognosis to 1000 mb. is [2]

$$\frac{d}{dt}(\overline{Z}_0-Z_0+G+kZ_T)=0, \tag{2}$$

where the subscript zero refers to the 1000-mb. level, k is a parameter (assumed constant) which depends on the mesh size, static stability, and the Coriolis parameter, and Z_T is the 1000–500-mb. thickness. Equation (2) is derived from the vorticity and thermodynamic energy equations under the assumptions of geostrophic, adiabatic, and frictionless motion. Also assumed are a functional relationship of the vertical velocity with height, with zero vertical velocity at the surface (level ground), and a straightline wind hodograph between 1000 mb. and 500 mb. The equation may be regarded as a special form of the potential vorticity theorem.

3. METHODS OF OBTAINING EXACT SOLUTIONS

The local change of $\overline{Z}_5-Z_5+G$ in (1) is obtained graphically by advecting this quantity the desired time interval in the geostrophic flow corresponding to the $(\overline{Z}_5+G)$-field and graphically subtracting the later field from the initial. Thus,

$$\Delta(\overline{Z}_5-Z_5+G)=-A_5, \tag{3}$$

where A_5 is the result of the subtraction. Therefore,

$$\Delta\overline{Z}_5-\Delta Z_5+A_5=0. \tag{4}$$

This equation can be solved either by means of a series expansion (Petterssen [4]) or by the relaxation method. The height change is then added to the initial height to obtain the predicted height.

In a similar manner the quantity $(\overline{Z}_0-Z_0+G+kZ_T)$ in equation (2) is advected with the geostrophic wind corresponding to the $(\overline{Z}_0+G+kZ_T)$-field to give an advective change, A_0. Thus

$$\Delta(\overline{Z}_0-Z_0+G+kZ_T)=-A_0, \tag{5}$$

or, upon substitution of Z_5-Z_0 for Z_T,

$$\Delta\overline{Z}_0-(1+k)\Delta Z_0+k\Delta Z_5+A_0=0. \tag{6}$$

Since ΔZ_5 is known from the results of (4), equation (6) can be solved for ΔZ_0 by either of the two methods mentioned above.

4. A METHOD OF OBTAINING SHORT, APPROXIMATE SOLUTIONS

To carry through all the operations implicit in the solution of equation (6) is a tedious and time-consuming job. However, on the basis of several years of experience with the method, both in research studies and classroom exercises, a number of short cuts have been developed which greatly shorten the procedure and which do not appear to have adverse effects on the accuracy of the forecasts.

When use is made of a 6° lat. mesh size, as originally suggested by Fjørtoft, variations of the quantity G in equation (1) are found to be small compared with variations of $\overline{Z}_5-Z_5$. Thus (1) may be simplified to

$$\frac{d}{dt}(\overline{Z}_5-Z_5)=0, \tag{7}$$

and the $\overline{Z}_5$ field alone determines the advecting wind. A further simplification results from measuring advective changes only at fixed grid points rather than continuously over the chart. With the help of a displacement ruler, the upstream value of $\overline{Z}_5-Z_5$ is advected to the point and is subtracted from the current value at the point to give A_5. In performing the advection four-fifths, rather than the total, geostrophic wind is used. This factor has been determined empirically and presumably compensates for the neglect of G and for a deviation, in the mean, of the level of nondivergence from the assumed level of 500 mb. Next it is assumed that $\Delta\overline{Z}_5$ is negligible aside of ΔZ_5 so that equation (4) may be written

$$\Delta Z_5=A_5. \tag{8}$$

Since the advection at a single grid point may be performed very rapidly and since the advections may be divided among a number of persons, a considerable saving of time is realized by making measurements at individual points. To take advantage of a team effort a master chart with grid points superimposed and duplicating facilities are required.

Once the 500-mb. height changes are computed, isopleths are drawn for use in the next step.

Next equation (2) is simplified by noting that the variations in $\overline{Z}_0$ and G are generally small compared with those in Z_0 and kZ_T. Thus (2) reduces to

$$\frac{d}{dt}(KZ_5-Z_0)=0, \tag{9}$$

where $K=k/(1+k)$ and Z_5-Z_0 has been substituted for Z_T. It can be shown that the advecting wind in (9) is now the geostrophic wind corresponding to KZ_5 (Estoque [1]). Consequently

$$\begin{aligned}\frac{\partial}{\partial t}(KZ_5-Z_0)&=-K\mathbf{V}_{g5}\cdot\nabla(KZ_5-Z_0)\\&=K\mathbf{V}_{g5}\cdot\nabla Z_0.\end{aligned} \tag{10}$$

Integration of (10) over the forecast interval, assuming no time variation of $\mathbf{V}_{g5}$, gives

$$K\Delta Z_5-\Delta Z_0=Z_{0i}-Z_{0u} \tag{11}$$

where Z_{0i} represents the initial 1000-mb. height at a point and Z_{0u} the value upstream which is advected to the point during the forecast interval.

Denoting now the predicted 1000-mb. height by Z_{0p} and noting that

$$Z_{0p}=Z_{0i}+\Delta_0, \tag{12}$$

we obtain from substitution of (11)

$$Z_{0p}=Z_{0i}+(K\Delta Z_5+Z_{0u}-Z_{0i})=Z_{0u}+K\Delta Z_5. \tag{13}$$

Since a typical value for K at middle latitudes is 1/2 and since this is a convenient number which gives good results in practice, we write as the final prediction formula,

$$Z_{0p}=Z_{0u}+\frac{1}{2}\Delta Z_5. \tag{14}$$

Equation (14) states the interesting result that the 1000-mb. height (or surface pressure) may be predicted by displacing (steering) the surface pressure pattern with a fraction (one-half) of the 500-mb. wind and adding a fraction (one-half) of the 500-mb. height change, a result which has long been known, at least in part, from empiricism. Thus both upper-level steering and reflection of height changes from aloft are shown to be important in the behavior of surface pressure systems. The upper-level change is particularly important in the problem of development and attests to the importance of the advection of vorticity at high levels in the deepening of surface Lows, as discussed by Petterssen [5] and others.

The final steps in obtaining the forecast consist of advecting 1000-mb. heights to the grid points (using one-half the geostrophic velocity at 500 mb.), analyzing the new pressure field, and then graphically adding the field of 500-mb. height change. The half factor is taken account of by relabeling the height changes with one-half values before addition.

The use of grid points, as before, has the advantage of allowing the advections to be divided among several workers. This advantage in speed would be offset by a loss in accuracy if an advective change were determined in this stage, rather than an advected pressure. Unless an exceedingly fine grid is used, important features of the pressure field are lost when changes at grid points are analyzed and added to the initial field. However, by advecting and analyzing pressures, the forecaster can be certain that these features are maintained. He visually affirms that the preliminary pressure analysis truly represents a displaced version of the original chart. The addition of the upper-level changes usually results in only minor modifications of this preliminary pressure field, though these modifications are important in that they determine the deepening and filling of the surface pressure systems, as mentioned before.

5. SUMMARY OF THE SHORT PROCEDURE

It is assumed that the field of 500-mb. height change is available either from the graphical method or preferably from the more exact solution of the barotropic vorticity equation transmitted by the National Weather Analysis Center.

Step 1. Superimpose surface and 500-mb. charts on a map containing grid points. Surface isobars should be drawn at 4-mb. intervals (approximately equivalent to a 100-ft. contour interval), 500-mb. contours at 200-ft. intervals, and grid points at about a 4° lat. separation. Prepare duplicate copies.

Step 2. By application of a geostrophic displacement scale to the 500-mb. contours, determine the surface pressure at the appropriate point *upstream* from the grid point and record this pressure at the grid point. If duplicate charts have been prepared, this work may be shared by a team of workers. Because of the one-half factor, it is important to note that the 200-ft. contour interval is treated as only a 100-ft. interval in applying the geostrophic wind scale. The measurements should be made along contour channels, using a flexible ruler, and allowance should be made for variations of speed along the channels.

Step 3. Analyze the preliminary prognostic pressure field making sure that the analysis represents a displaced version of the initial pressure distribution. Take special care with positions of high and low centers and frontal troughs.

Step 4. Graphically add the 500-mb. height changes to the preliminary pressure chart to give the final prognostic chart. If surface isobars are at 4-mb. intervals, height changes should be at 200-ft. intervals (because of the one-half factor).

Experience has suggested a few modifications of the prognostic chart which generally lead to improved forecasts. These will be discussed in section 7.

6. SOME SAMPLE FORECASTS

In order to demonstrate the quality of the forecasts obtained by the foregoing method the results of 20 predictions of surface pressure made during July 1959 are presented in figure 1. The prognostic charts were prepared by students, under the guidance of the instructors, in a special course for Air Force officers held during the summer session at University of California at Los Angeles. Also shown in the figure are the corresponding prognostic charts received by facsimile from the National Weather Analysis Center and the verification charts. Missing days are due to weekends and variations in the laboratory routine, not to a selection of cases.

In comparing the prognostic charts based on the graphical method with the subjective prognoses from the Analysis Center, it is important to note that the latter are for a somewhat longer time period (30 hours) and therefore may appear to be of poorer quality than the graphical prognoses even though they possess greater skill. With this proviso in mind, it would appear from examination of

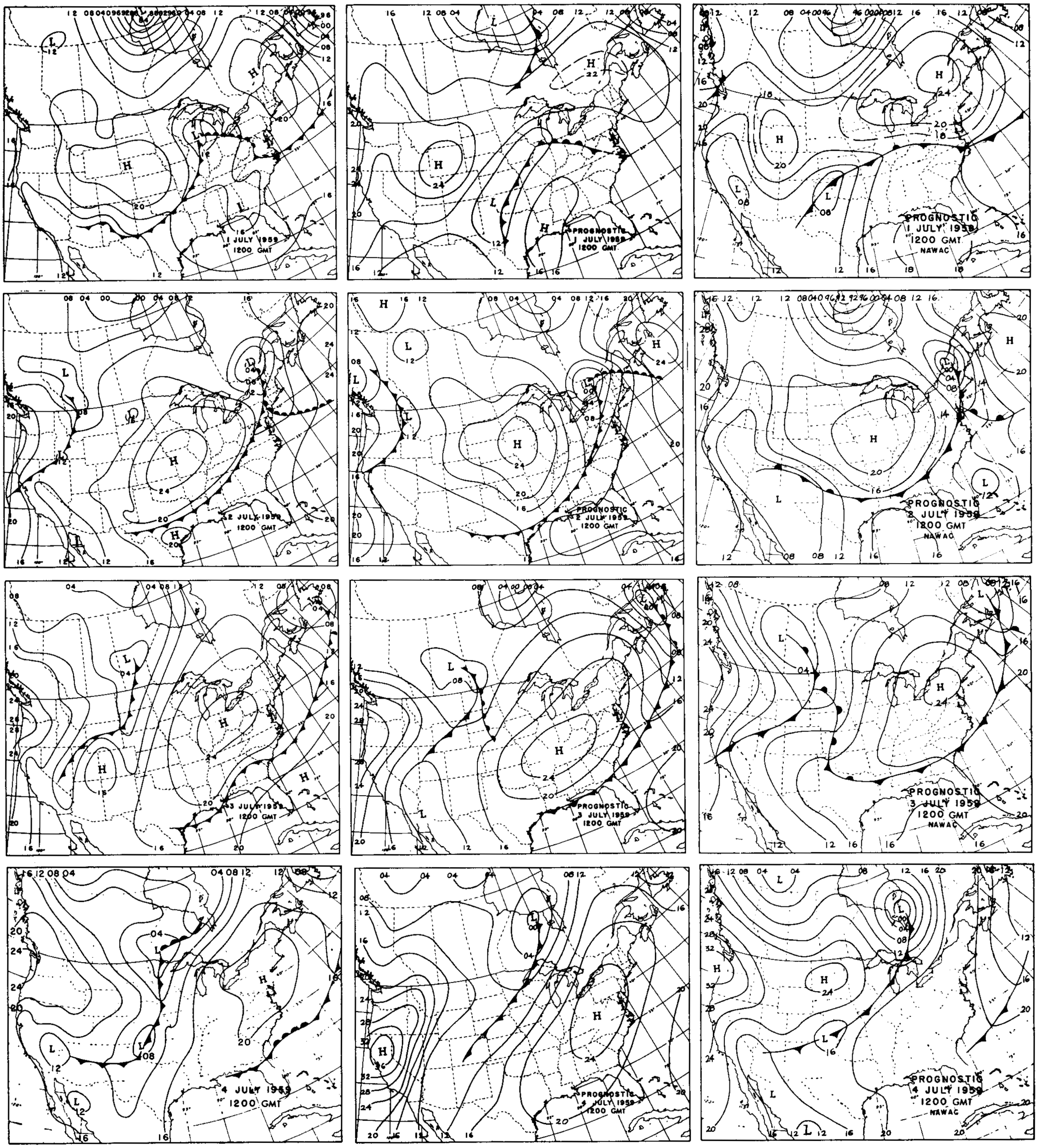


FIGURE 1.—Interrupted series of surface charts for period July 1 to August 1, 1959. Left, observed chart; center, 24-hr. prognostic chart, Reed method; right, 30-hr. prognostic chart, National Weather Analysis Center.

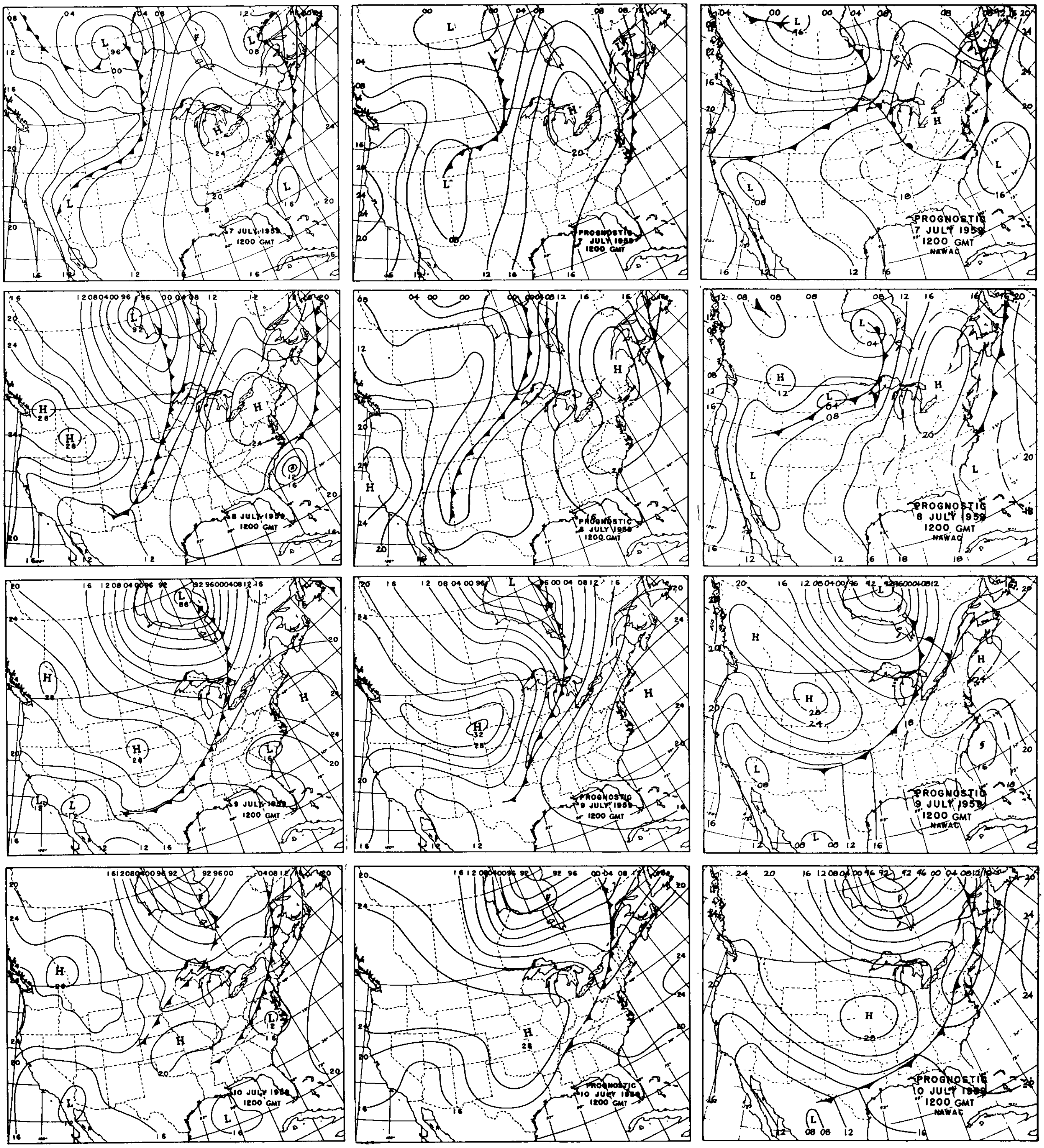
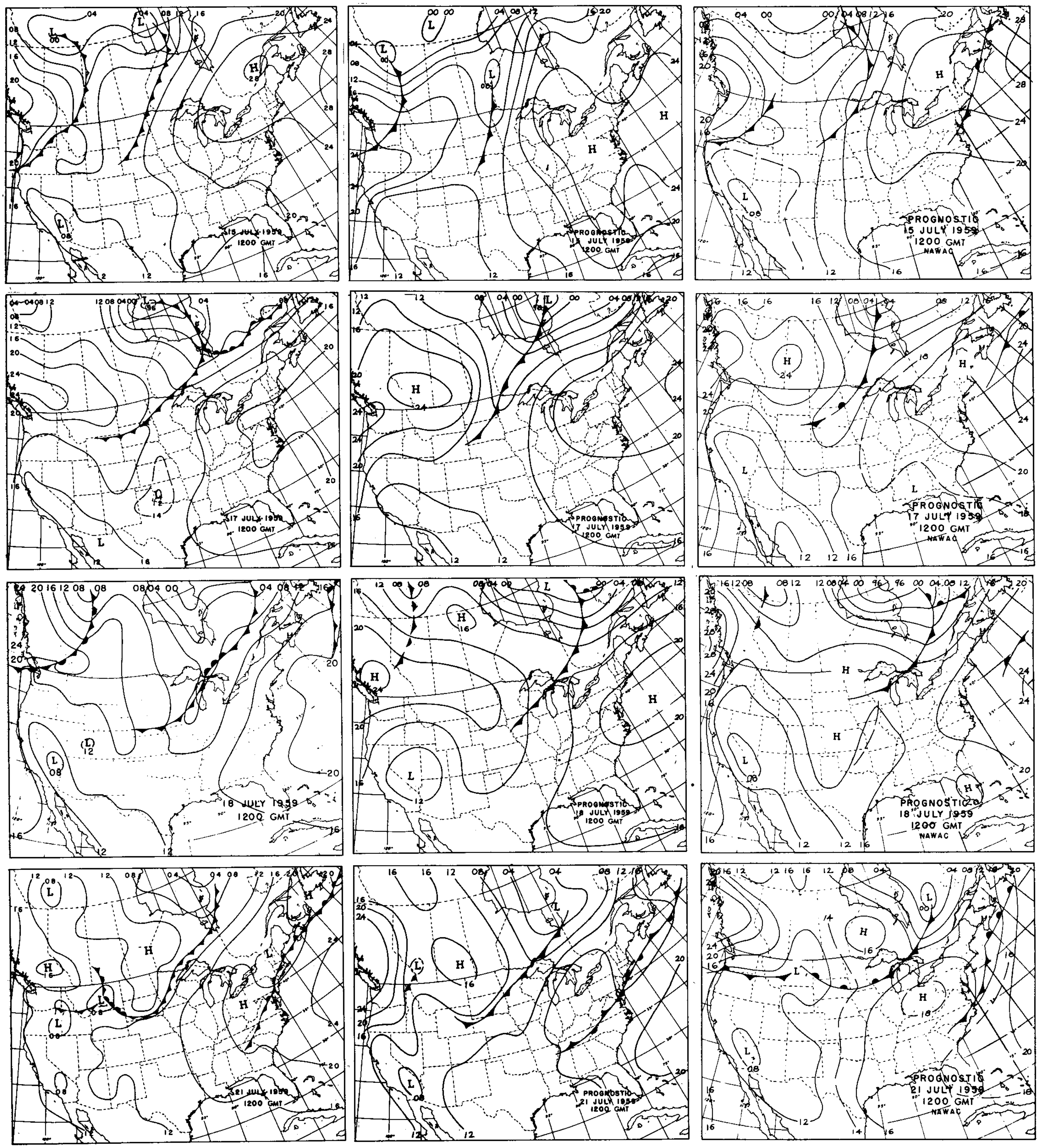


FIGURE 1.—Continued.



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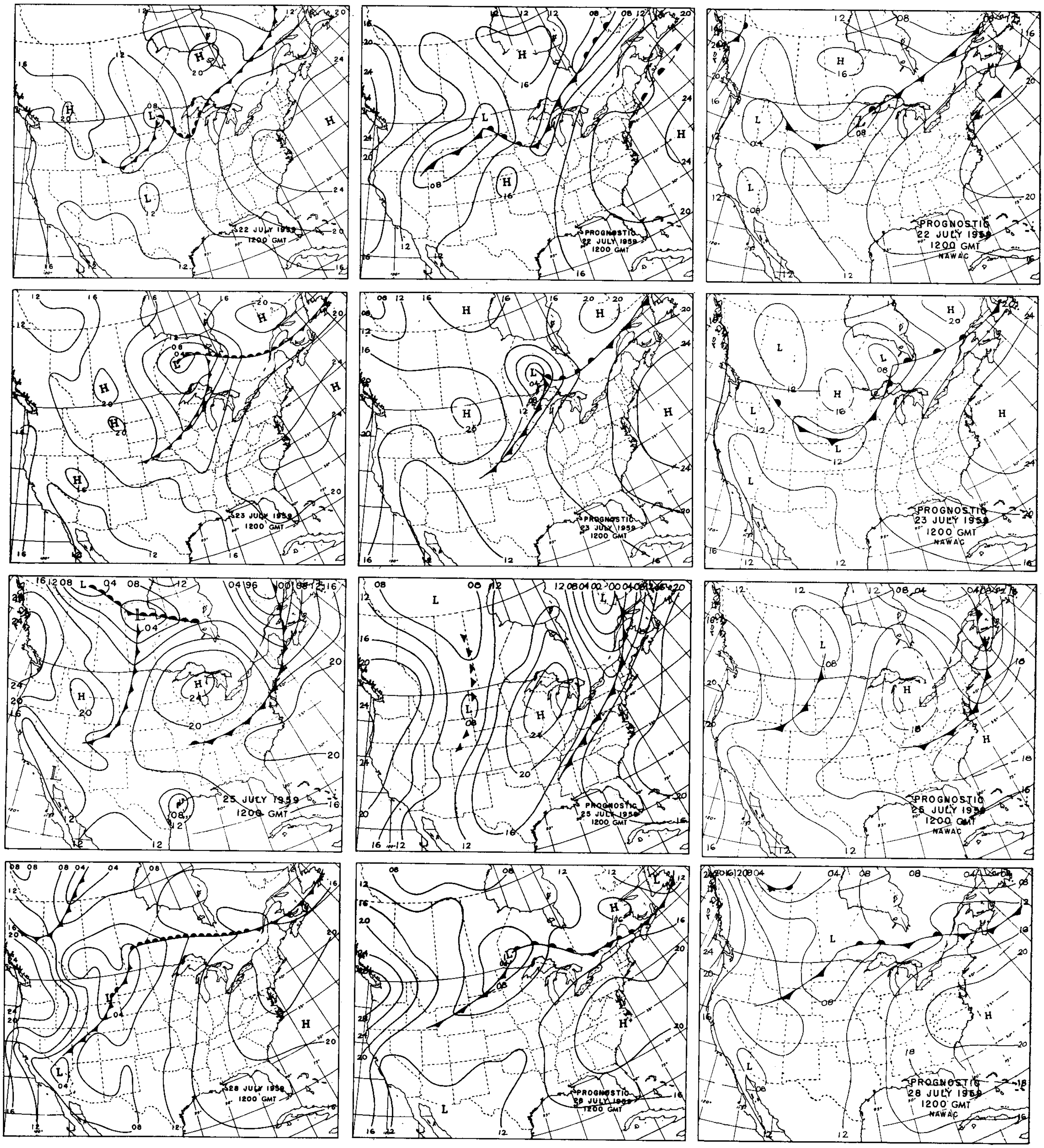

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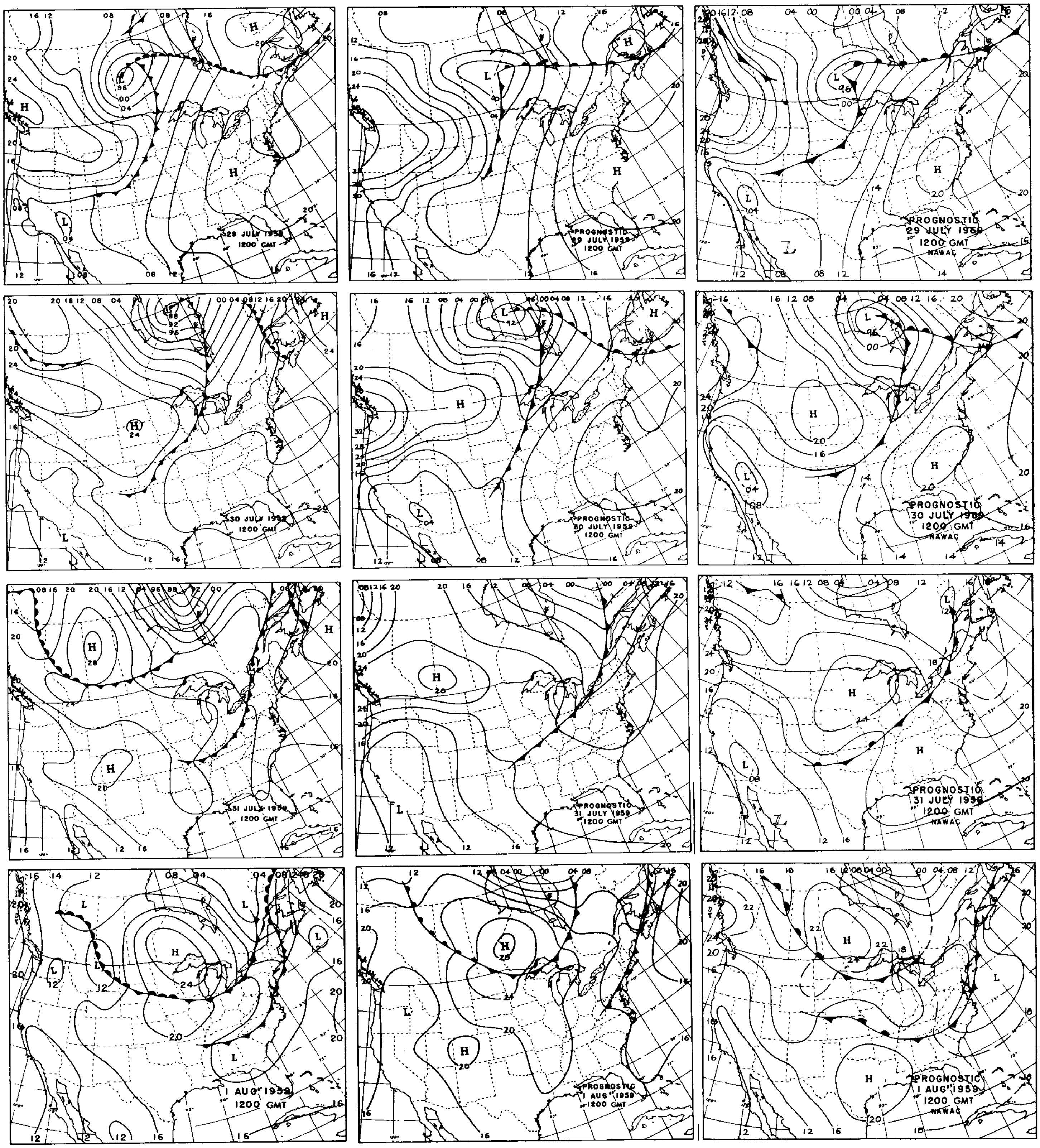

FIGURE 1.—Continued.

the charts that there is little difference in the accuracy of the two types of forecasts. On most days the prognoses bear a fairly close resemblance to each other and also to the verification chart. Where errors in the displacements of cold and occluded fronts or low centers occur, almost always the subjective forecasts show an overdisplacement and the graphical forecasts the contrary.

Concerning the forecasting of development, it appears that the graphical method is superior, though the difference in the forecast interval may be important in this respect. The Low which developed near Lake Michigan on July 1 was missed completly by the subjective method, but was clearly indicated in the graphical prognosis. This Low resulted from the downward reflection of relatively large height falls at 500 mb., and its development was even more apparent at intermediate stages in the preparation of the forecast than on the prognostic chart.

The filling of the original low center near southern Hudson Bay and the formation of a new Low north of Minnesota on July 4 was hinted at on the graphical prognosis but not on the subjective. On July 8, the subjective forecast, contrary to the graphical, showed a new low center over North Dakota. From the verification chart it is apparent that this feature failed to materialize. On the other hand on July 21, the subjective forecaster did a better job of predicting the wave development over Montana, though its presence was evident on the graphical prognosis.

In a few cases bad features of the forecasts were traced to student errors—for example, the poor position of the warm front in New England on July 2 and the overdisplacement of the low center in southern Canada on July 28. Because of emphasis on large-scale features no attempt was made to predict the hurricane of July 7–10 in the eastern United States.

7. FURTHER COMMENTS AND SUGGESTIONS FOR IMPROVEMENT

On the basis of rather extensive experience with the graphical method, both in synoptic laboratories and in research studies, the following observations may be made concerning the performance of the method and possible ways of improvement.

1. The performance is better in summer than in winter probably as a result of the greater persistence and regularity of long-wave features in summer and the less rapid and extreme development of short-wave systems.

2. It is possible to allow for the effect of mountains in generating lee troughs and crest ridges by procedures suggested by Estoque [6] and Haltiner and Hesse [7]. In the short method described here the effect is equivalent to adding to the 500-mb. steering field an additional geostrophic wind field, determined by a fraction of the topographical contours. Thus surface pressure systems tend to acquire an additional southward component of movement on the east sides of mountain ranges and an opposite movement on the west.

No attempt was made to take orographic influences into account in the present series of forecasts, but qualitative comparison of the actual and prognostic charts leaves little doubt that the forecasts would have been consistently improved by incorporating the effect of orography.

3. The development of new waves is usually too weakly predicted. A typical example is the case of July 1, 1959. Since cyclogenesis is characteristically overemphasized in machine integrations using short time steps, it appears that this feature is connected with the long time interval employed in the graphical integration.

4. Along most of their extents cold and occluded fronts are displaced with one-half the 500-mb. wind, as required by the model. In the vicinity of cols, however, where the upper-level flow often parallels the surface front, the movement appears to be governed more by frictional outflow at low levels. In such cases the cold air advances generally at a speed of about 5 knots despite the absence of a geostrophic component normal to the front. This empirically derived fact was used in preparing the prognostic charts and was the only empirical correction applied. The relationship of warm fronts to upper-level flow is more erratic, probably as a result of the tendency of their lower portions to flatten and become retarded in certain situations.

5. The most serious shortcoming of the graphical method is the use of a single large time step in the integration. In other words, as applied here, the method makes no allowance for the change in steering current during the forecast period. It is believed that the underdisplacement of fronts in this series was almost entirely due to this cause.

Because of the usual location of surface Lows under a more or less straight current aloft, the change in upper-level flow generally does not lead to serious errors in their displacements during periods of 24 hours or less. However, surface Highs oftentimes are situated just to the rear of upper-level troughs. In such cases the one-step integration carries the High around the trough, while in retrospect the movement of the trough may be sufficient to prevent the High from advancing beyond the trough line. The positions of the Highs on the forecasts for July 3 and 9 to the north of the observed positions may be accounted for by this type of error.

Since the 500-mb. prognostic chart is prepared independently, it is possible to correct the advection of the surface pressure systems in the light of the changes in the flow pattern aloft. As yet no tests have been conducted making use of the prognostic 500-mb. chart, but the impression is that such use would almost always lead to improved forecasts.

In view of the success of the 500-mb. forecast issued by the Joint Numerical Weather Prediction Unit, it would be of interest to use this in conjunction with the current 500-mb. chart in displacing Highs, Lows, and fronts, and other significant features of the surface chart. The numerically predicted 500-mb. height changes would then be used to modify the preliminary estimate obtained from displacing the surface features. Experience with the graphical method would lead one to believe that highly useful forecasts may be obtained in this way.

It would also be desirable to arrive by statistical means at a better estimate of the coefficient for displacing surface pressure systems and reflecting down the 500-mb. height changes. The value used here (0.5) has been selected partly for convenience and does not necessarily represent the best value.

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